

# Flow Field Analysis Near Bend of U-shaped Tube

Nimisha Yadav

Department of Applied Science And Humanities  
K N I T Sultanpur up

**Abstract**— In this paper we have carried out an oscillatory laminar flow analysis near bend of curved tube. Womersley numbers are used to conduct the experiment and the secondary flow induced in the cross section was analysed by a solid tracer method. In the paper, oscillatory flows is analyzed in straight tube and curved shaped tube.

**Keywords:** Oscillatory flow, Curved tube, Secondary flow, Solid Tracer, Womersley number.

## 1. INTRODUCTION

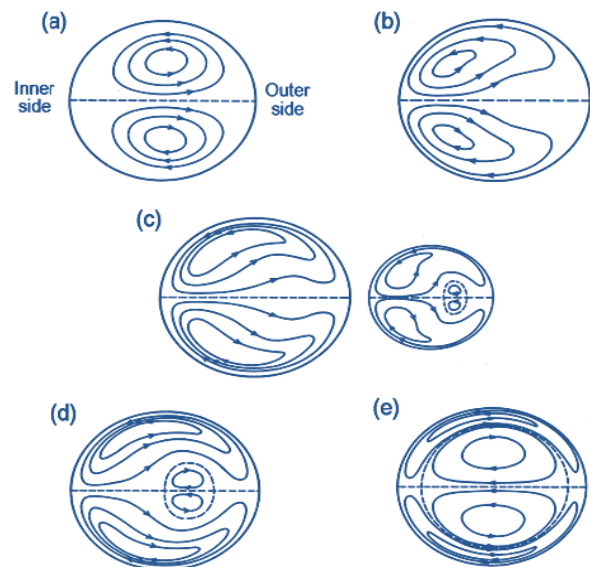
The purpose of this paper is to analyse the flow field near the bend entrance of curved tube. To this end, we perform a visualization experiment and velocity measurement on both the steady streaming and the secondary flow motion induced in the horizontal and vertical planes, respectively.

The method used is a type of multi-exposure solid-tracer method using a stroboscopic light. By this method, one can readily and simultaneously obtain not only the pattern but also the velocity information of the secondary-flow field. First, this method was applied to a fully developed oscillatory flow in a circular curved tube with a curvature radius ratio  $Rc$  of 7.6 [10], where  $Rc=R/a$  is the ratio of the curvature radius of the tube axis  $R$  to the radius of the tube  $a$ . The important results obtained were that the oscillatory flow is mainly governed by the Womersley number  $a$  and the Dean number  $D$ , and that the pattern of the secondary flow formed in the cross section can be classified into five patterns including Lyne circulation, as shown in Figure 1. Here, the Womersley number is a non-dimensional frequency parameter and is defined as  $a=\omega/\nu^{1/2}$  with  $\omega$  being the angular frequency of oscillation and  $\nu$  the kinematic viscosity of the fluid. The Dean number is expressed as  $D=ReRc^{-1/2}$ , where  $Re=2aw_{m,o}/\nu$  is the Reynolds number with  $w_{m,o}$  being the amplitude of the instantaneous axial velocity  $w_m$  averaged over the cross section. The Womersley number can be physically interpreted as the square root of the ratio of the unsteady inertia force to the viscous force, while the Dean number is the ratio of the square root of the product of the inertia and centrifugal forces to the viscous force.

The secondary flow in the entrance region is predicted to interfere with axial flow development, changing it in a complicated manner. Few studies have been reported on the oscillatory entrance flow in a curved tube. Bertelsen and

Thorsen [11] and Naruse et al. [12] considered axial flow development. In the study of Bertelsen and Thorsen [11], profiles of the axial and outward velocities in a symmetrical horizontal plane were merely examined.

This paper is concerned with the development of the secondary flow motion generated in the vicinity of a curved tube with a straight section connected to each end of the curved tube. In particular, we examine a high Womersley-number flow problem, in which the secondary flow shows a peculiar pattern, i.e., Lyne circulation. Subsequently, we discuss the effects of the governing parameters  $a$ ,  $D$  and  $Rc$  on the development of the secondary flow. Moreover, we describe the relationship between the secondary flow development and the additional steady streaming essentially formed in the horizontal plane.

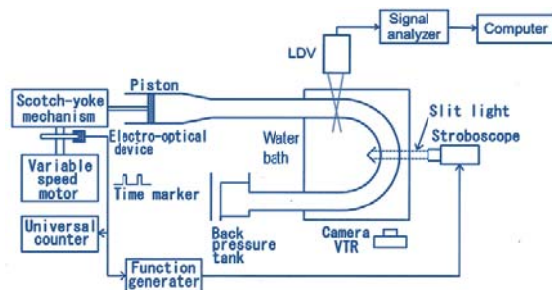


**Figure 1. Secondary flow patterns induced in the cross section in the fully developed region, as classified by Sudo et al. [10]. (a) Dean circulation. (b) Deformed Dean circulation. (c) Intermediate circulation between Dean and Lyne circulation. (d) Deformed Lyne circulation. (e) Lyne circulation.**

## 2. PROCEDURE

### 2.1. Experimental Apparatus

The setup used to perform experiment is shown in Figure 2. Setup comprises of an oscillatory flow generator, a test tube, and visualizing and measuring devices. An oscillating water flow for experiment is generated by piston pump and the desired volume-cycled flow rate  $Q$  was adjusted by the selection of the diameter (6, 10 and 15 mm) and stroke (0-60 mm) of the piston, and by adjusting the rotational frequency of a variable speed motor. The instantaneous flow rate  $Q(t)$  is given by  $Q = Q \sin \Theta$

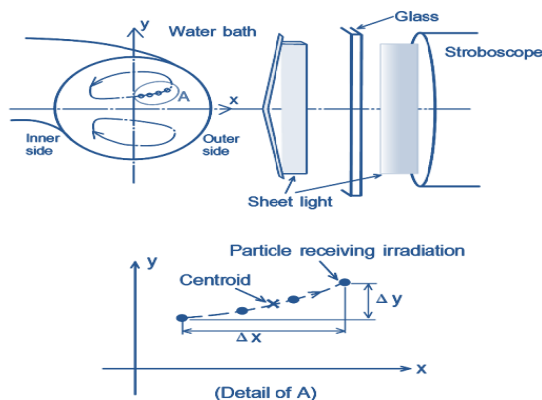


**Figure 2. Schematic diagram of experimental apparatus. The apparatus consists of an oscillatory flow generator, a test tube, and visualizing and measuring devices.**

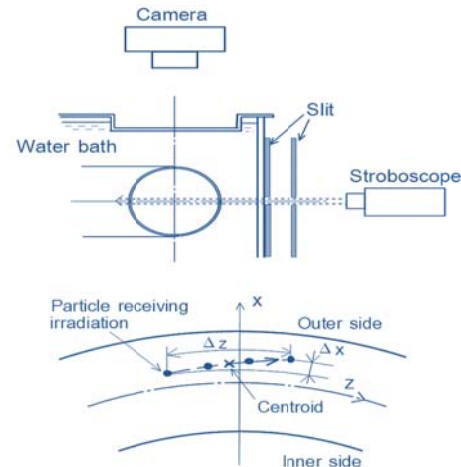
### 2.2. Flow Visualization and Velocity Measurement

Visualization experiments were conducted for both the secondary stream and the steady streaming.

The flow of fluid was rendered by a solid tracer method. Relation between the layers of fluid of the secondary stream and steady stream and the stroboscope and the camera, respectively were shown in Figure 3 and 4. This method was explained by Sumida et al. [9] and Sudo et al. [10]. When the Womersley number is high, the fluid in the tube flows and returned back with the same phase as the phase of reciprocating piston pump, and the axial flow velocity becomes uniform, except near the walls of the tube.



**Figure 3. Visualization and velocity measurement of secondary stream.**



**Figure 4. Visualization and velocity measurement of steady stream**

Firstly, measurements of the secondary stream in the cross section were performed at four stream wise stations between  $z=-d$  in the upstream tube and  $\Omega=30^\circ$  in the curved tube. Sheet light with a thickness of 2-3 mm was applied to the cross section ( $x$ - $y$  plane) perpendicular to the tube axis, as shown in Figure 3. The irradiation was repeated 4 to 20 times. Moreover, this visualization technique was also applied to the steady stream in the horizontal plane shown in Figure 6. The velocity information obtained from the steady stream generated in the symmetrical plane should be useful in understanding the development of the secondary stream in the cross section.

We considered the obtained Lagrangian velocity is Eulerian at its corresponding position. Thus, the velocity calculated in this manner is time-averaged over one cycle. Also, it denotes the magnitudes of the secondary stream and the steady stream in the horizontal plane.

The displacements of the tracer particles ( $\Delta x$ ,  $\Delta y$ ) and ( $\Delta x$ ,  $\Delta z$ ) in the vertical and horizontal planes, respectively, were determined using a personal computer. Here,  $\Delta x$ ,  $\Delta y$  and  $\Delta z$  are the components of the vectors in the  $x$ ,  $y$  and  $z$  directions, respectively (Figures 3 and 4). Then the corresponding time-averaged velocities  $u_{ta}$ ,  $v_{ta}$  and  $w_{ta}$  at the centroid of the path line elements were calculated easily from  $\Delta x/\Delta t$ ,  $\Delta y/\Delta t$  and  $\Delta z/\Delta t$ , respectively. Here, the suffix  $ta$  indicates time-averaged values and  $\Delta t$  is the time required for the particle displacement, i.e.,  $\Delta t = (m-1)\delta t$  with  $m$  being the number of irradiations.

The total measurement error was estimated to be less than 7% except near the tube wall. In addition, the error for the secondary stream was large compared with that for the steady streaming, particularly in the straight section far from the bend entrance.

### 3. RESULTS

According to the findings of Sudo et al. [10], for low Womersley numbers, the time and cross-sectional averaged velocity  $S_{ta,m}$  of the secondary flow increases in proportion to the Dean number. On the other hand, when the Womersley number exceeds a specific value, the secondary flow velocity  $S_{ta,m}$  decreases sharply and then changes in proportion to  $D^2/\alpha^{2.5}$ .

At high values of the characteristic parameter  $\beta(\alpha^2/D)$ , i.e.,  $\beta > 0.655$ , unsteady inertia effects predominate in the core region of the curved tube, while viscous effects are restricted to a thin layer adjacent to the tube wall. In the cross section, an additional pair of vortices rotating in the opposite direction to the primary vortices is generated in the core region. Therefore, two pairs of vortices are formed in the cross section of the curved tube. This secondary flow pattern is well known as Lyne circulation [5].

In the following section, we consider the oscillatory flow for a  $\beta$  value considerably higher than 0.655, at which the secondary flow pattern is divided roughly into two. Furthermore, we discuss the effects of the Womersley number on the Lyne-type secondary flow near the entrance of the curved tube. Also, we reveal the features of the oscillatory entrance flow.

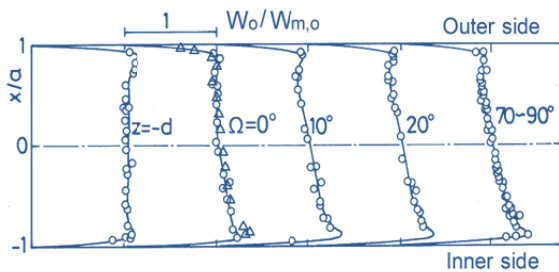


Figure 5. Axial-velocity amplitude  $w_o$  along the tube axis [ $\beta=3.84$  ( $\alpha=24, D=150, Rc=4$ )].

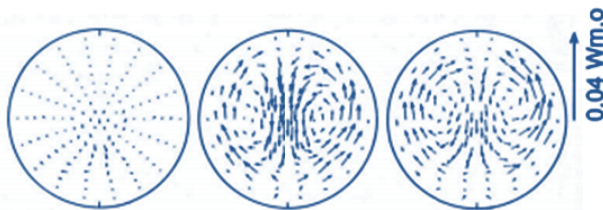


Figure 6. Velocity vectors of secondary stream near the bend entrance [ $\beta=1.08$  ( $\alpha=18, D=300, Rc=10; \delta t=T$ )].



Figure 7. Steady streaming in horizontal plane ( $D=300, Rc=10$ ).

### 3.3. Strengths of Secondary Stream and Steady Streaming

Comparing the secondary stream in the cross section with the steady streaming in the horizontal plane in Figures 6 and 7, respectively, we can clearly observe the transition between the straight-tube and curved-tube oscillatory flows. The flow mechanism of the secondary stream and the steady streaming is shown in Figure 8. The steady streaming in the inner part of the horizontal plane immediately upstream from the entrance is toward the inner-wall side. The vortex motion of the secondary stream begins to collapse in the outer part of the cross section, while in the curved section, the steady streaming in the horizontal plane changes its direction to the centripetal direction with an increase in  $\Omega$ .

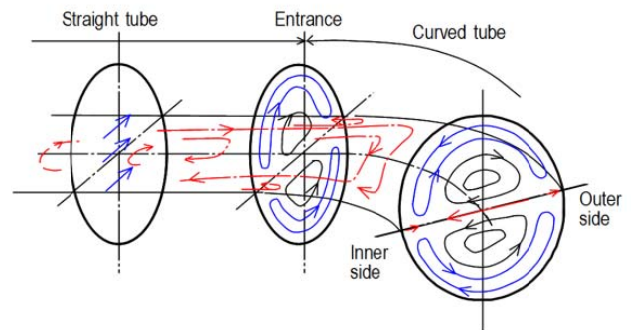


Figure 8. Schematic diagram showing the relation between the secondary stream and the steady streaming near the bend entrance. Blue and black lines show the secondary stream in the cross section, and red lines denote the steady streaming in the horizontal plane.

To estimate the strengths of the secondary stream and the steady streaming, we introduce the quantities  $|w_{ta}|_x$  and  $S_{ta,m}$ , respectively. Here,  $|w_{ta}|_x$  is the average of  $|w_{ta}|$  over the  $x$ -axis,  $|w_{ta}|$  being the absolute value of the time-averaged axial velocity  $w_{ta}$  of the steady streaming.  $S_{ta,m}$  denotes the cross-sectional average value of  $S_{ta}$ , which is the secondary stream velocity described by  $S_{ta} = (u_{ta}^2 + v_{ta}^2)^{1/2}$ .

Figure 9 shows the changes in  $|w_{ta}|_x$  and  $S_{ta,m}$  along the tube axis. For  $\alpha=18$  ( $D=300$ ), the strength  $S_{ta,m}$  of the secondary stream is slightly larger at the bend entrance but does not reach 1% of  $w_{m,o}$ , while the change in  $S_{ta,m}$  in the curved section of the tube is small. On the other hand,  $|w_{ta}|_x$ , denoting the strength of the steady streaming in the horizontal plane, is largest in the section of  $z \approx -1d$ , at which the center of the streaming is located. However, it decreases further away from the center. Furthermore, both the secondary stream and the steady streaming weaken with an increase in  $\alpha$ , since the unsteady inertia force increases so that the change in the  $w_o$  profile in the  $\Omega$  direction decreases. For higher  $\beta$  ranging from 10.4 to 32.0, Bertelsen[8] measured the time-averaged secondary-stream velocity  $u_{ta}$  on the  $x$ -axis in a fully developed region with  $Rc=9.26$ . Regarding the average of  $|u_{ta}|$  over the  $x$ -axis estimated from his data,  $|u_{ta}|/w_{m,o}$  gives a value of  $(0.3-0.7) \times 10^{-3}$ . The value is approximately one-tenth of that in the present data for  $\beta=1.08-3$ , appearing to be of an inverse ratio of  $\beta$ .

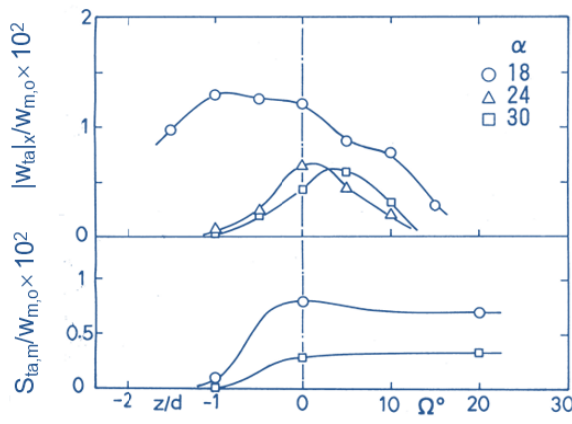


Figure 9. Strengths of secondary stream and steady streaming for  $D=300$  and  $Rc=10$ . The upper and low figures show the steady streaming and secondary stream, respectively.

Figure 10 shows the values of secondary stream and steady streaming in strongly curved tube. According to the experiments in reference [11],  $|u_{ta}|/w_{m,o}$  is estimated to be  $0.32 \times 10^{-2}$  at the station of  $z=-0.0955d$ , for  $\alpha=21.8$  and  $D=308$ , in the curved tube with  $Rc=4$ . This supports to our results. As the Dean number decreases,  $|w_{ta}|_x$  decreases.

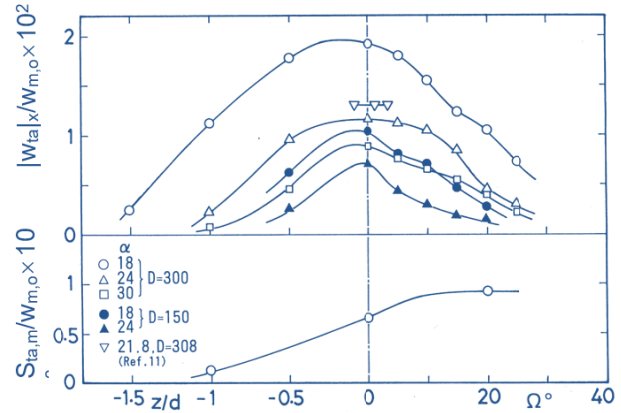


Figure 10: Strengths of secondary stream and steady streaming in strongly curved tube ( $Rc=4$ ).

#### 4. CONCLUSIONS

Flow visualization experiments were performed for the oscillatory entrance flow in curved tubes. The important findings drawn from the present results for a high characteristic parameter,  $\beta > 0.655$ , are as follows:

1. The classification of the secondary-flow pattern for the fully developed oscillatory flow is applicable to the flow developing downstream from the bend entrance. Two pairs of vortices, the so-called Lyne circulation, appear even at the bend entrance.
2. The strength of the secondary stream developing in the curved section does not change markedly and is roughly equal to that in the fully developed region.
3. For  $Rc=10$ , the secondary stream in the straight section decays near the outer and subsequently inner walls. For  $Rc=4$ , i.e., a strong curvature, the secondary motion in the plane of  $z \approx -1d$  shifts from inward to outward in the central part of the cross section.
4. Accompanying such secondary-stream transitions, in the horizontal plane, a steady streaming is generated with a double structure. Furthermore, the trajectory of the fluid streaming drifts and forms an elliptical region with an increase in  $\alpha$ , with the strength at the bend entrance about two times as large as that of the secondary stream.

---

**REFERENCES**

- [1] S. A. Berger, L. Talbot and L. -S. Yao, "Flow in curved pipes," *Annual Review of Fluid Mechanics*, vol. 15, pp. 461-512, 1983.
- [2] M. Sumida, "Pulsatile entrance flow in curved pipes: effect of various parameters," *Experiments in Fluids*, vol. 43, pp. 949-958, 2007.
- [3] B. Timité, C. Castelain and H. Peerhossaini, "Pulsatile viscous flow in a curved pipe: effects of pulsation on the development of secondary flow," *International Journal of Heat and Fluid Flow*, vol. 31, pp. 879-896, 2010.
- [4] M. Jarrahi, C. Castelain and H. Peerhossaini, "Secondary flow patterns and mixing in laminar pulsating flow through a curved pipe," *Experiments in Fluids*, vol. 50, pp. 1539-1558, 2011.
- [5] W. H. Lyne, "Unsteady viscous flow in a curved pipe," *Journal of Fluid Mechanics*, vol. 45, no. 1, pp. 13-31, 1970.
- [6] R. G. Zalosh and W. G. Nelson, "Pulsating flow in a curved tube," *Journal of Fluid Mechanics*, vol. 59, pp. 693-705, 1973.
- [7] B. R. Munson, "Experimental results for oscillating flow in a curved pipe," *Physics of Fluids*, vol. 18, no. 12, pp. 1607-1609, 1975.
- [8] A. F. Bertelsen, "An experimental investigation of low Reynolds number secondary streaming effects associated with an oscillating viscous flow in a curved pipe," *Journal of Fluid Mechanics*, vol. 70, pp. 519-527, 1975.
- [9] M. Sumida, K. Sudo and H. Wada, "Pulsating flow in a curved pipe: secondary flow," *JSME International Journal*, Ser II, vol. 32, no. 4, pp. 523-531, 1989.
- [10] K. Sudo, M. Sumida and R. Yamane, "Secondary motion of fully developed oscillatory flow in a curved pipe," *Journal of Fluid Mechanics*, vol. 237, pp. 189-208, 1992. [11] A. F. Bertelsen and L. K. Thorsen, "An experimental investigation of oscillatory flow in pipe bends," *Journal of Fluid Mechanics*, vol. 118, pp. 269-284, 1982.
- [12] T. Naruse, Y. Nishina, H. Kugenuma and K. Tanishita, "Developing oscillatory flow in a strongly curved tube," *Transactions of Japan Society of Mechanical Engineers*, vol. 56B, no. 529, pp. 2562-2569 (in Japanese), 1990.
- [13] S. Uchida, "The pulsating viscous flow superposed on the steady laminar motion of incompressible fluid in a circular pipe," *Zeitschrift für Angewandte Mathematik und Physik*, vol. 7, pp. 403-422, 1956.
- [14] D. P. Telionis, *Unsteady Viscous Flows*. New York: Springer, p. 154, 1981.
- [15] M. Sumida and K. Sudo, "Oscillatory flow in curved pipes, 4th report: velocity distribution in entrance region," *Transactions of Japan Society of Mechanical Engineers*, vol. 60B, no. 569, pp. 63-70 (in Japanese), 1994.